

## Quadratic Functions

A. Transform each function  $y = ax^2 + bx + c$  into the form  $y = a(x + p)^2 + q$

$$y = ax^2 + bx + c$$

$$y - c = ax^2 + bx$$

$$y - c = a\left(x^2 + \frac{b}{a}x\right)$$

Process:  $y - c + \frac{b^2}{4a} = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right)$

$$y + \frac{b^2 - 4ac}{4a} = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right)$$

$$y = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) - \frac{b^2 - 4ac}{4a} \Rightarrow y = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a}$$

p or h

q or k

## Assignment

$$y = 2x^2 + 5x - 7$$

$$y + 7 = 2\left(x^2 + \frac{5}{2}x\right)$$

1.  $y + 7 + \frac{25}{8} = 2\left(x^2 + \frac{5}{2}x + \frac{25}{16}\right)$

$$y + \frac{7 \cdot 8 + 25}{8} = 2\left(x + \frac{5}{4}\right)^2$$

$$y + \frac{81}{8} = 2\left(x + \frac{5}{4}\right)^2$$

$$y = 2\left(x + \frac{5}{4}\right)^2 - \frac{81}{8}$$

$$y = 5x^2 + x - 3$$

$$y + 3 = 5\left(x^2 + \frac{1}{5}x\right)$$

2.  $y + 3 + \frac{1}{20} = 5\left(x^2 + \frac{1}{5}x + \frac{1}{100}\right)$

$$y + \frac{3 \cdot 20 + 1}{20} = 5\left(x + \frac{1}{10}\right)^2$$

$$y + \frac{61}{20} = 5\left(x + \frac{1}{10}\right)^2$$

$$y = 5\left(x + \frac{1}{10}\right)^2 - \frac{61}{20}$$

$$y = -3x^2 - 7x + 1$$

$$y - 1 = -3\left(x^2 + \frac{7}{3}x\right)$$

$$y - 1 - \frac{49}{12} = -3\left(x^2 + \frac{7}{3}x + \frac{49}{36}\right)$$

3.  $y + \frac{-1 \cdot 12 - 49}{12} = -3\left(x + \frac{7}{6}\right)^2$

$$y - \frac{61}{12} = -3\left(x + \frac{7}{6}\right)^2$$

$$y = -3\left(x + \frac{7}{6}\right)^2 + \frac{61}{12}$$

$$y = -7x^2 + 4x - 1$$

$$y + 1 = -7\left(x^2 - \frac{4}{7}x\right)$$

$$y + 1 - \frac{16}{28} = -7\left(x^2 - \frac{4}{7}x + \frac{16}{196}\right)$$

4.  $y + \frac{1 \cdot 28 - 16}{28} = -7\left(x - \frac{4}{14}\right)^2$

$$y + \frac{12}{28} = -7\left(x - \frac{4}{14}\right)^2$$

$$y = -7\left(x - \frac{4}{14}\right)^2 - \frac{12}{28}$$

$$y = x^2 + x - 3$$

$$y + 3 = (x^2 + x)$$

$$y + 3 + \frac{1}{4} = \left(x^2 + x + \frac{1}{4}\right)$$

$$5. \quad y + \frac{3 \cdot 4 + 1}{4} = \left(x + \frac{1}{2}\right)^2$$

$$y + \frac{13}{4} = \left(x + \frac{1}{2}\right)^2$$

$$y = \left(x + \frac{1}{2}\right)^2 - \frac{13}{4}$$

$$y = \frac{1}{2}x^2 - 3x + 5$$

$$2y = x^2 - 6x + 10$$

$$2y - 10 = (x^2 - 6x)$$

$$2y - 10 + \frac{36}{4} = \left(x^2 - 6x + \frac{36}{4}\right)$$

$$7. \quad 2y + \frac{-10 \cdot 4 + 36}{4} = \left(x - \frac{6}{2}\right)^2$$

$$2y - \frac{4}{4} = \left(x - \frac{6}{2}\right)^2$$

$$2y = \left(x - \frac{6}{2}\right)^2 + \frac{4}{4}$$

$$y = \frac{1}{2} \left(x - \frac{6}{2}\right)^2 + \frac{4}{8}$$

$$y = \frac{1}{2}(x - 3)^2 + \frac{1}{2}$$

$$y = -x^2 + 2x - 5$$

$$y + 5 = -1(x^2 - 2x)$$

$$y + 5 - \frac{4}{4} = -1 \left(x^2 - 2x + \frac{4}{4}\right)$$

$$6. \quad y + \frac{5 \cdot 4 - 4}{4} = -1 \left(x - \frac{2}{2}\right)^2$$

$$y + \frac{16}{4} = -1 \left(x - \frac{2}{2}\right)^2$$

$$y = -1 \left(x - \frac{2}{2}\right)^2 - \frac{16}{4} \Rightarrow y = -1(x - 1)^2 - 4$$

$$y = -\frac{1}{3}x^2 + 2x - 7$$

$$3y = -x^2 + 6x - 21$$

$$3y + 21 = -1(x^2 - 6x)$$

$$3y + 21 - \frac{36}{4} = -1 \left(x^2 - 6x + \frac{36}{4}\right)$$

$$8. \quad 3y + \frac{21 \cdot 4 - 36}{4} = -1 \left(x - \frac{6}{2}\right)^2$$

$$3y + \frac{48}{4} = -1 \left(x - \frac{6}{2}\right)^2$$

$$3y = -1 \left(x - \frac{6}{2}\right)^2 - \frac{48}{4}$$

$$y = -\frac{1}{3} \left(x - \frac{6}{2}\right)^2 - \frac{48}{12}$$

$$y = -\frac{1}{3}(x - 3)^2 - 4$$